WOLLO UNIVERSITY
KIOT

SCHOOL OF MECHANICAL AND CHEMICAL ENGINERING

MECHATRONICS 3RD YEAR

REGULATION AND CONTROL(McEng 3174)

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Chapter one

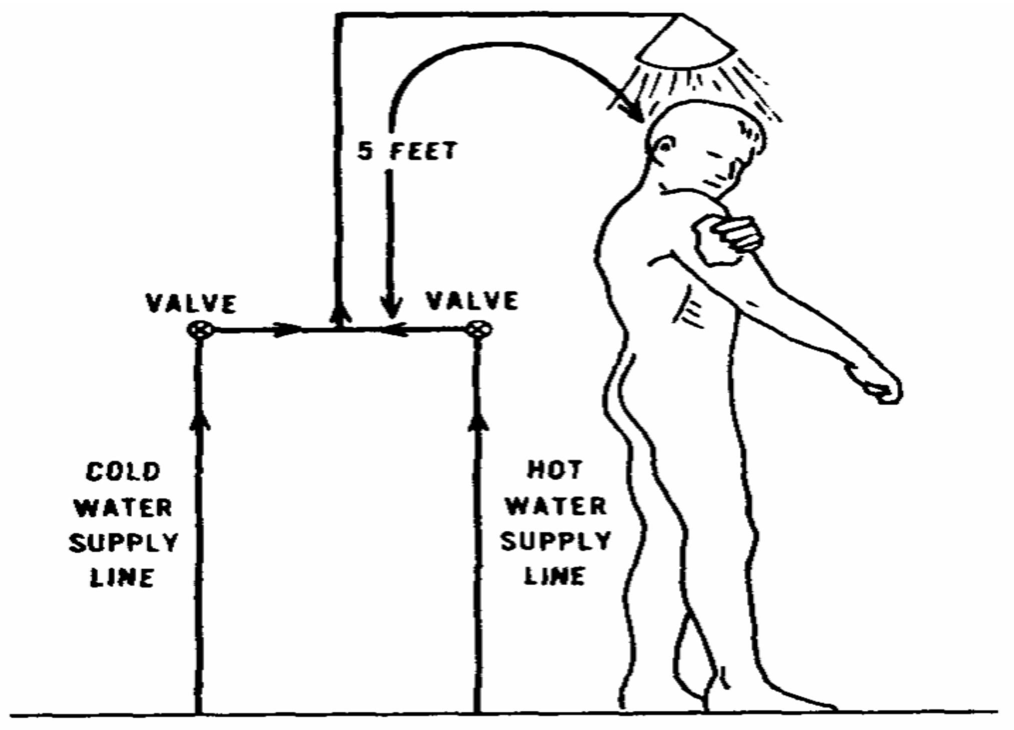
Equations and Models of Linear Systems

Contents:

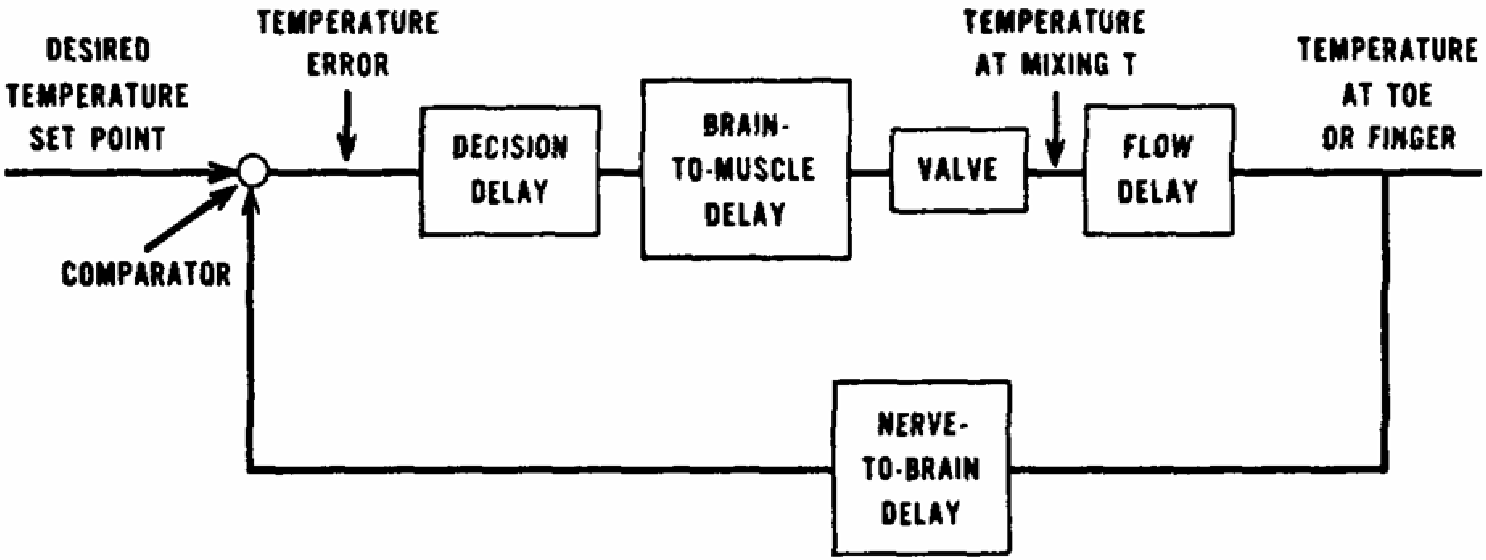
- Introduction
- Block diagrams
- Transfer function
- Block diagram reduction rules
- Mathematical modelling of Physical systems
- Signal flow graph

Introduction:

- The concept of control engineering was initially introduced for electrical and mechanical application.



Flow diagram for shower example.



Block diagram of shower control system.

- Automatic control has played a vital role in advanced engineering and science.
- It extremely important in:
  - Space vehicle system
  - Missile
  - Robotics
  - Modern Manufacturing & industries process)

What is system?

- When a number of elements or components are connected in a sequence to perform a specific function.

Input → **System** → **Out put**

(Excitation)
- Cause & effect r/ship
- Signal coming to the system

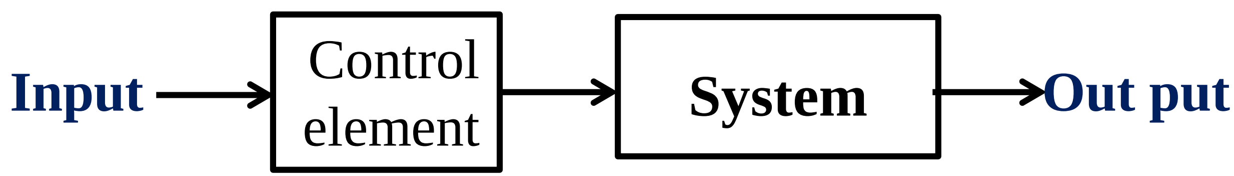
(Effect or response)

Eg. – car, fan, plants, etc.

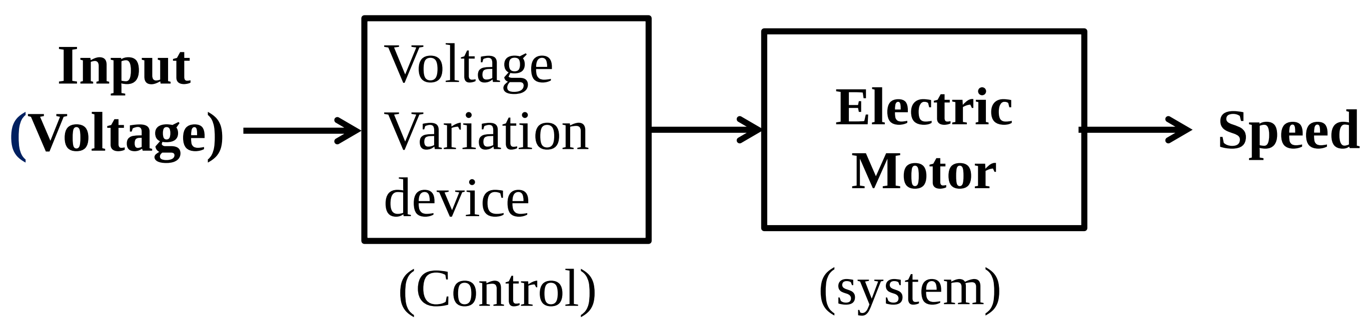
- System may be:
  - ➢ Single – input – single output (SISO)
  - ➢ More than one input (MIMO)

What is Control System?

- It is an arrangement of physical components connected in a proper manner to perform a specific function, ***in which the output is controlled by input.***



Eg. – Refrigerator, AC, power system, etc.



- **Application of Control system:**
  - Used in electrical system to control voltage, current
  - Used to control the position and acceleration of driving systems
  - Used to control or regulate the machine parameters like displacement, acceleration in mechanical systems
  - In medical fields to regulate temperature, pressure in medical instruments.
- Some basic terminologies must be defined before discuss control system.
  - **Controlled variable(CV)**
    - ✓ It is a quantity or condition that measured and controlled
  - **Manipulated variable (MV)**
    - ✓ The quantity/ condition that is varied by controller so as to affect the value of the controlled variable

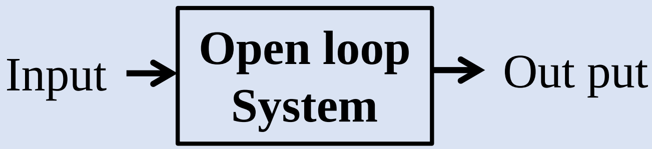
- **Plants:**
  - ✓ It may be a piece of equipment, perhaps just set of machine parts functioning together to perform a particular operation

    (Mechanical device, chemical reactor, cars, etc.)
- **Disturbance:**
  - ✓ Is the signal that tends to adversely affect the value of output of the system
  - ✓ It generated
    - Within the system (internal disturbance)
    - Outside the system; it is an external and called input
- **Feedback control:**
  - ✓ Refers to an operation, that in the presence of disturbance tends to reduce the difference b/n the o/p of the system and some reference input.

Basic types of control system *… Cont'd*

- In general there are two types of control system

i. Open loop system



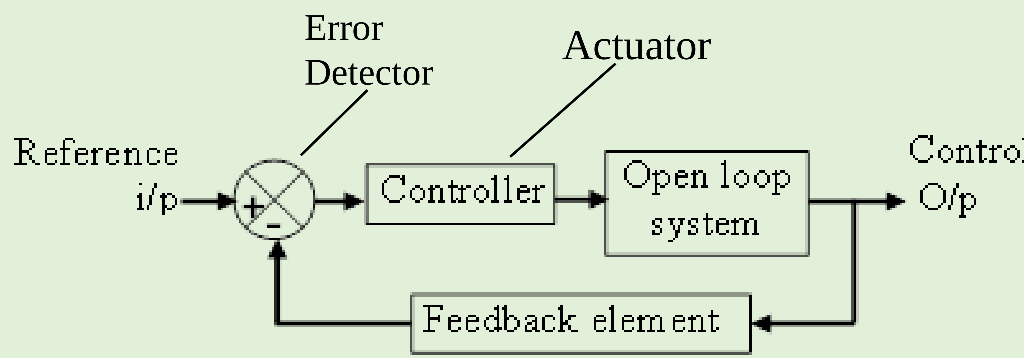
Advantages:

- Simple & economical
- Easier to construct (integrate)
- Stable

Disadvantage:

- In accurate
- Change in o/p are not corrected automatically.

ii. Closed loop system



Advantages:

- Accurate (reduce the disturbance)
- Less effected by noise

Disadvantage:

- Complex & costier.
- Feed back reduces the overall gains of the system
- Stability is the main problem
  (to get stable reduce sensibility factor)

- **Controller = error detector + control logic elements**

- Modern control system uses sensor and encoders as control system components
  - Potentiometers
  - AC/DC servo meters
  - Stepper motor
  - Tacho generators

- **Eg**. a. Traffic light controller (May be used OL or CL system)

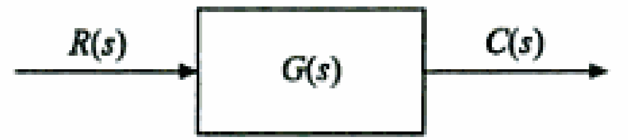
  b. Speed controller of car (may be open loop system)

  Desire Speed → Controller → Engine → Actual speed

- **Types of control system**
  - Linear system
  - Non linear system
  - Time variant system
  - Time invariant system
  - Linear time variant system
  - Linear time invariant system

Block Diagram

- It is a pictorial or symbolic representation of the ***cause and effect relationship b/n the in put and the out put of a control system.***

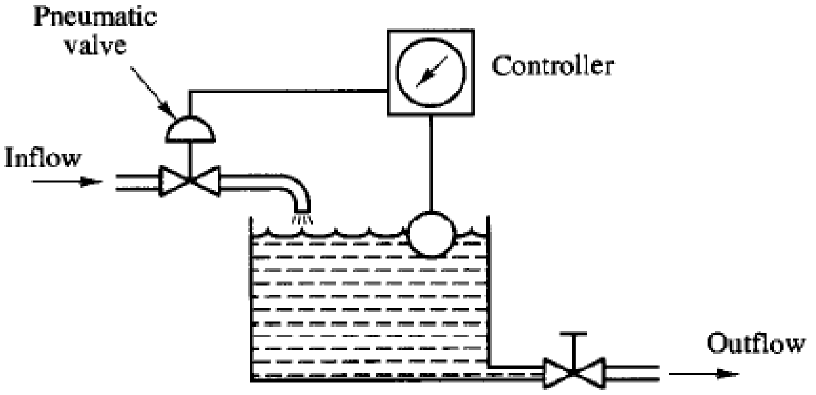


A Block Diagram

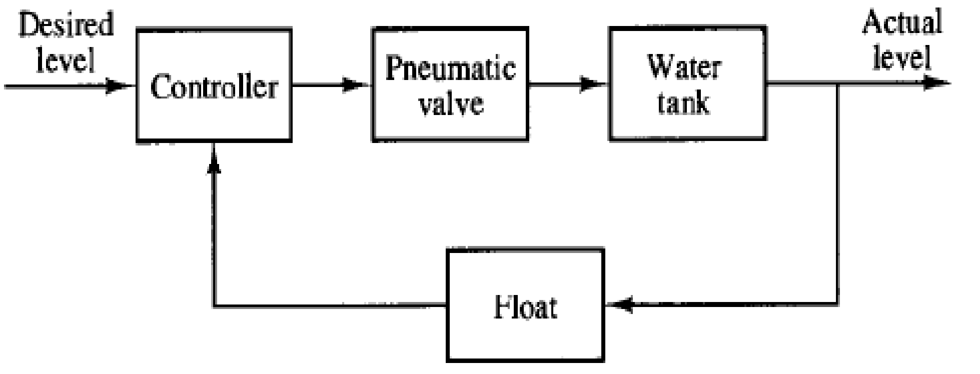
- The block diagram technique has been developed not only to show the interrelation ship b/n the constituents of a control system but also to do algebraic manipulations through their reduction procedure.
- **Advantage of block diagram**
  - Simple to construct
  - Function of each components or elements can easily visualized
  - Over all transfer function of a system can be calculated
  - Used to studies individual and over all performance of a components of a system
- **Disadvantages**
  - No information about the physical structure of a system is given by BD.
  - Source of energy is not shown in the BD.

Example

Figure (a) is a schematic diagram of a liquid-level control system. Here the automatic controller maintains the liquid level by comparing the actual level with a desired level and correcting any error by adjusting the opening of the pneumatic valve. Figure (b) is a block diagram of the control system. Draw the corresponding block diagram for a human-operated liquid-level control system.

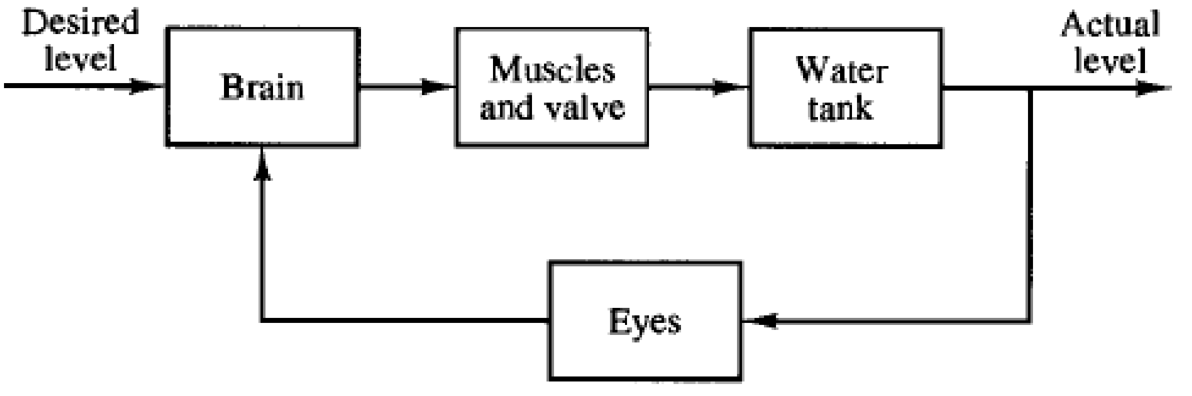


(a)



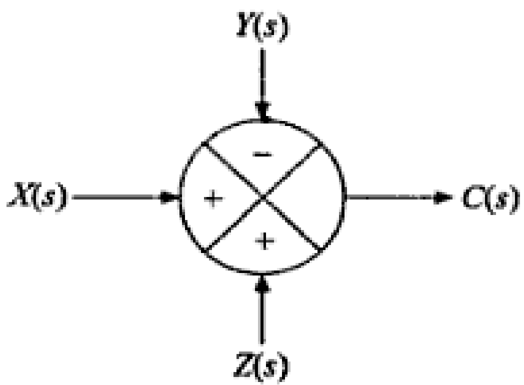
(b)

Block diagram of human-operated liquid-level control system.



Basic Element of Block Diagram

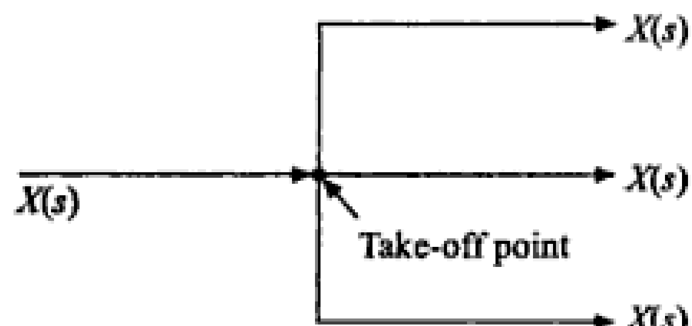
- Before proceed to do that, let us familiarize ourselves with a few definitions concerning block diagram representations in control systems.
- **Block:** series or parallel
- **Summing point:** represented by a circle with inner indications of signs (see figure) the points show the addition or subtraction of signals from different sources. [only one output].



➢ Here, the algebraic relation b/n the different variables at the summing point is

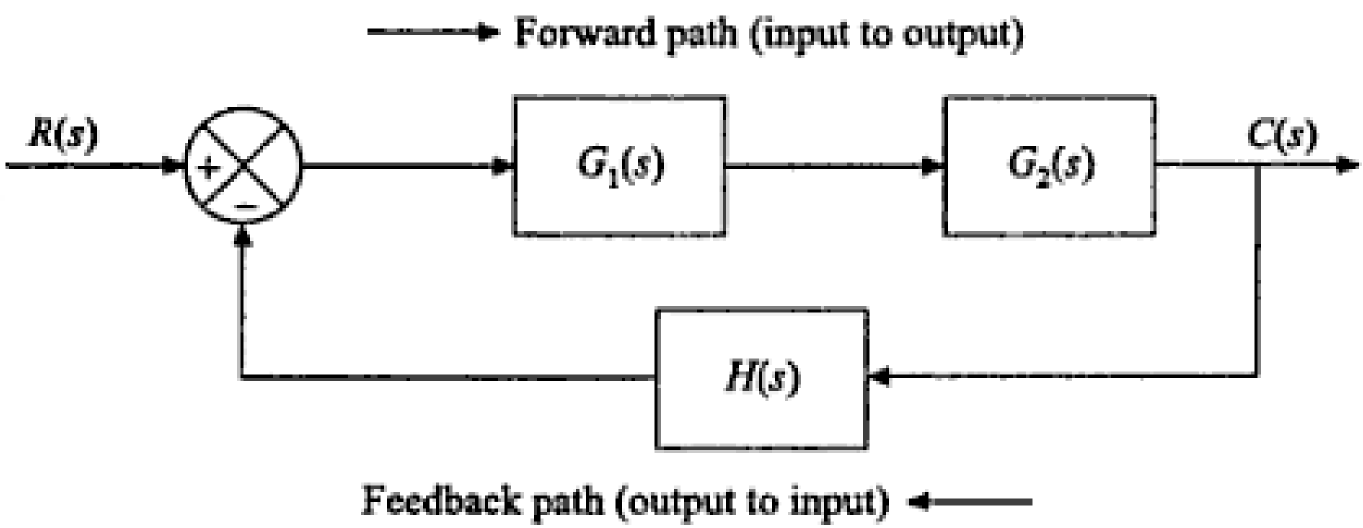
$C(s) = X(s) - Y(s) + Z(s)$

- **Take- off point (branching point):** If the output of a signal branches off from one point, it used to allow a signal to be used by two or more blocks or summing points.



Take-off point in a block diagram.

- **Branches:** are lines joining the block in a block diagram
- **Arrows:** indicate the direction of flow of signal in a system
- **Forward and feedback paths:** the paths are indicated in a block diagram given in fig. below. ***A forward path*** is indicated by an arrow pointing towards right while a left pointing arrow indicates a ***feedback path***.



Forward and feedback paths in a block diagram.

Transfer Function … *cont'd*

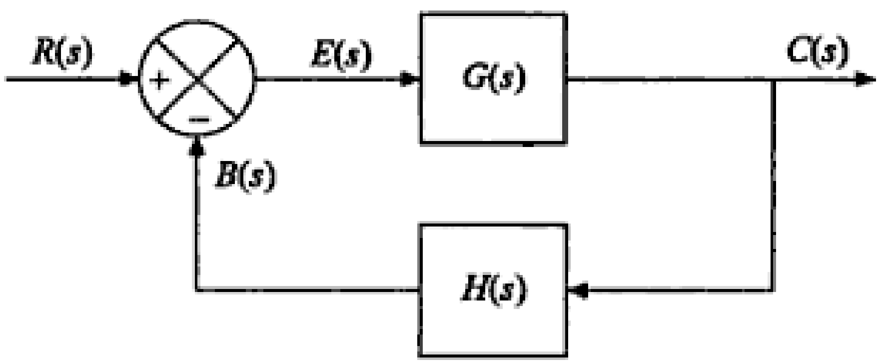
➢ Transfer function of a control system is the ratio of Laplace transform of output to Laplace transform of input.

i.e. $\text{Transfer Function} = \frac{L[Output\ Variable]}{L[Input\ Variable]}\Big|_{Zero\ initial\ conditions}$

▪ **Types of feed back**

➢ There are two types. Such as: *Negative and Positive feed backs*

i. Negative feed back *(when the o/p of the 2nd block oppose the reference i/p)*



List of symbols used in block diagrams

$R(s)$ = reference input
$C(s)$ = controlled output
$B(s)$ = feedback signal
$E(s)$ = signal that actuates control action
$G(s)$ = forward-path transfer function
$H(s)$ = feedback-path transfer function
$T(s)$ = closed-loop transfer function = $C(s)/R(s)$

Transfer Function of Negative feed back control loop system

From Figure we get

$$C(s) = E(s)G(s) \tag{1}$$

$$B(s) = C(s)H(s) \tag{2}$$

$$E(s) = R(s) - B(s) \tag{3}$$

Substituting Eq. (3) in Eq. (1), we get

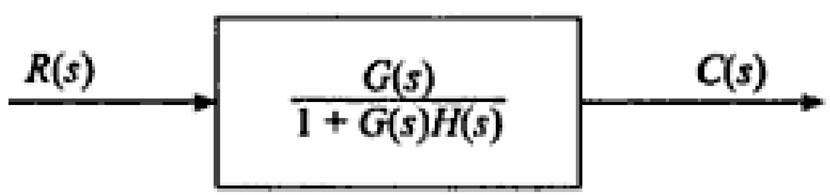
$$C(s) = [R(s) - B(s)]G(s) \tag{4}$$

Plugging the value of $B(s)$ from Eq. (2) in Eq. (4), we get

$$C(s) = R(s)G(s) - C(s)H(s)G(s) \tag{5}$$

Equation (5), on rearrangement, yields the closed-loop transfer function as

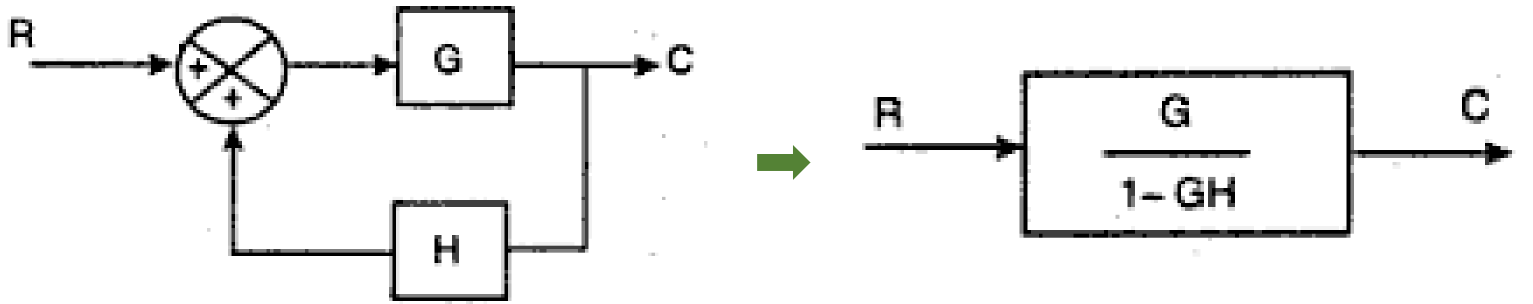
$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{6}$$



Reduced form of closed-loop of Figure

ii) Positive Feed back

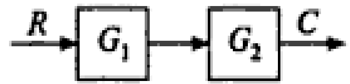
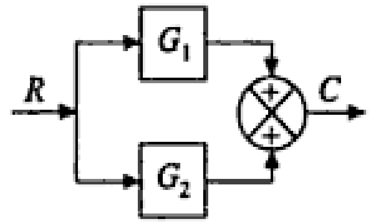
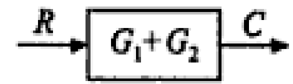
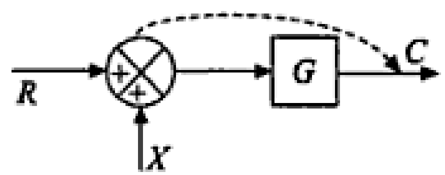
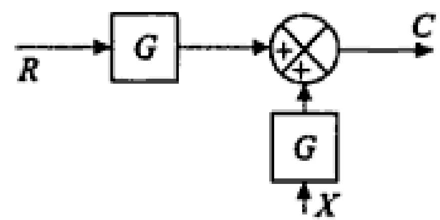
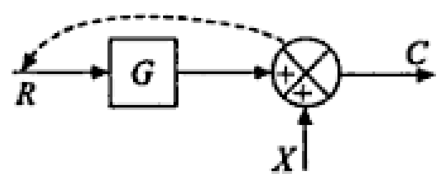
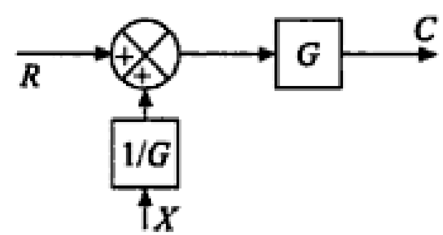
- ✓ If the output of the second block (feed back element) aid the ref – input the feed back is called positive feed back.



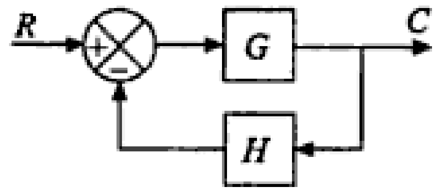
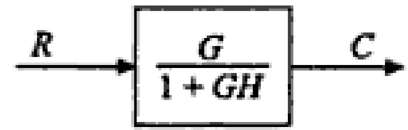
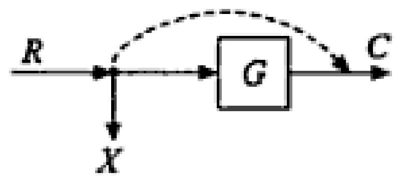
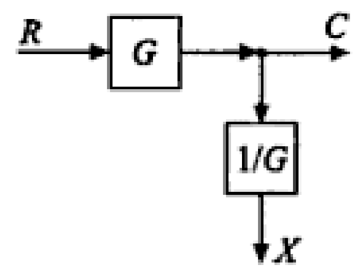
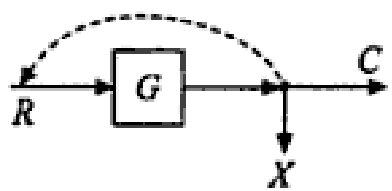
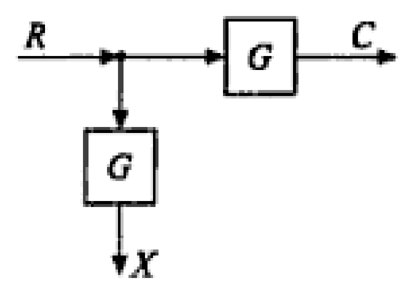
Effect of feedback

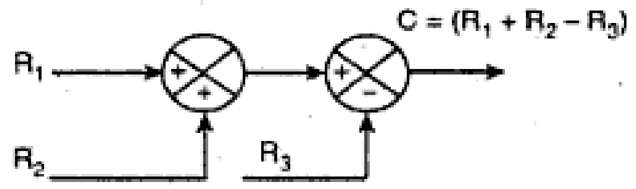
- ✓ On gain
- ✓ On sensitivity
- ✓ On stability (when output is more controllable)

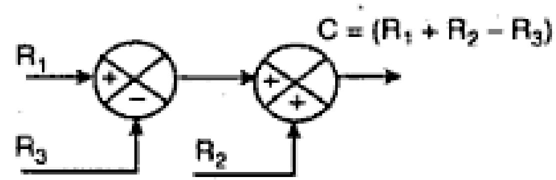
Block Diagram Reduction Rules

Transformation	*Original diagram*	*Equivalent diagram*
Combining blocks in cascade		
Combining blocks in parallel		
Moving a summing point beyond a block	 $C = (R + X)G$	 $C = RG + XG$
Moving a summing point behind a block	 $C = RG + X$	 $C = (R + X/G)G = RG + X$

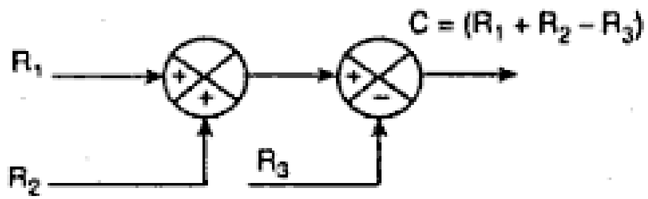
Cont..

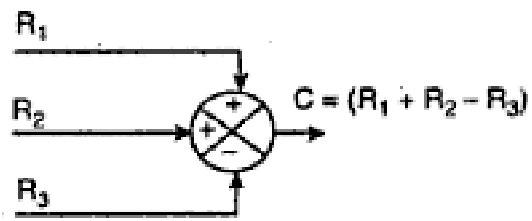
Transformation	*Original diagram*	*Equivalent diagram*
Eliminating a feedback loop		
Moving a take-off point beyond a block	 $X = R \quad C = RG$	 $X = R \quad C = RG$
Moving a take-off point behind a block	 $X = C = RG$	 $X = C = RG$



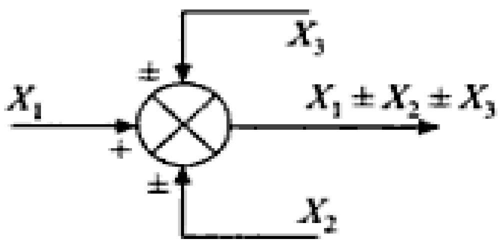


Interchanging of summers





Combining summers



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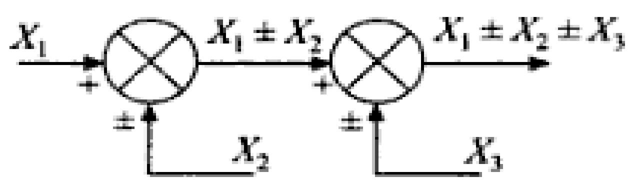


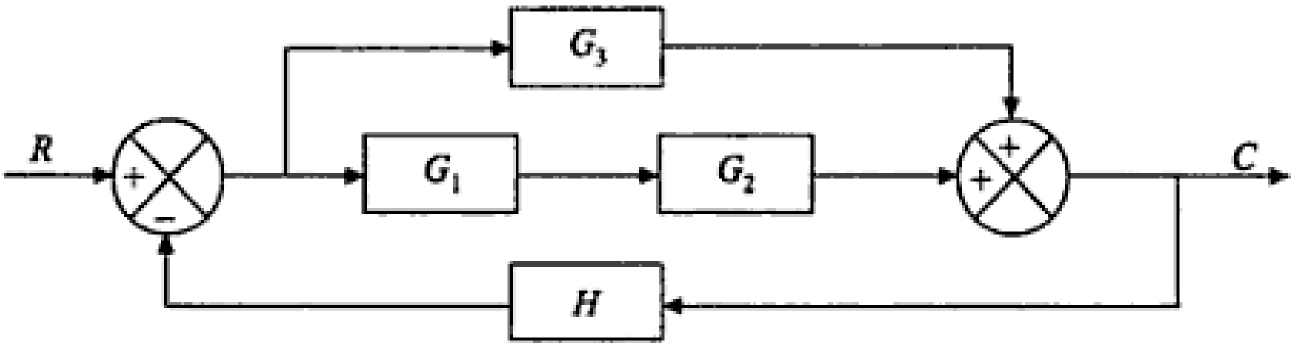
Splitting a summing point

It is better to follow a general procedure for block diagram reduction. The steps of the procedure are as follows:

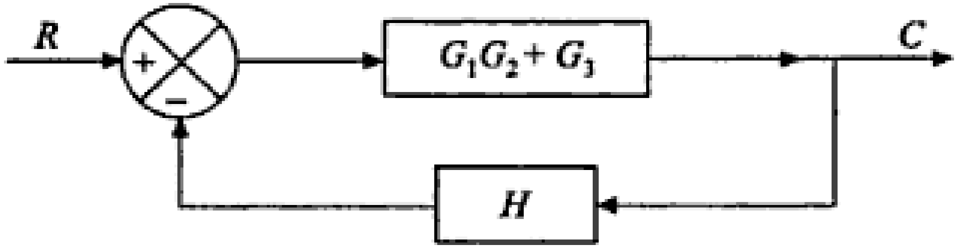
Step 1. Reduce cascade blocks.
Step 2. Reduce parallel blocks.
Step 3. Eliminate feedback loops.
Step 4. Move summing points to the right and take-off points to the left.
Step 5. Repeat steps 1 to 4 until a simple form is obtained.

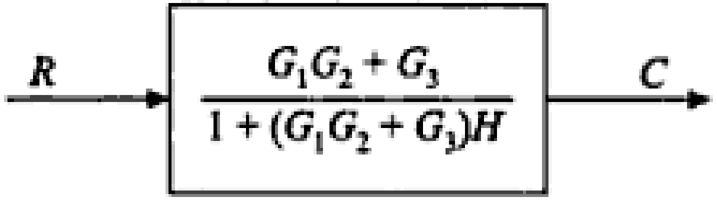
Determine the transfer function of the following block diagrams:

(1)



Solution: First we combine cascaded and parallel blocks. As a result, we get the block diagram as shown

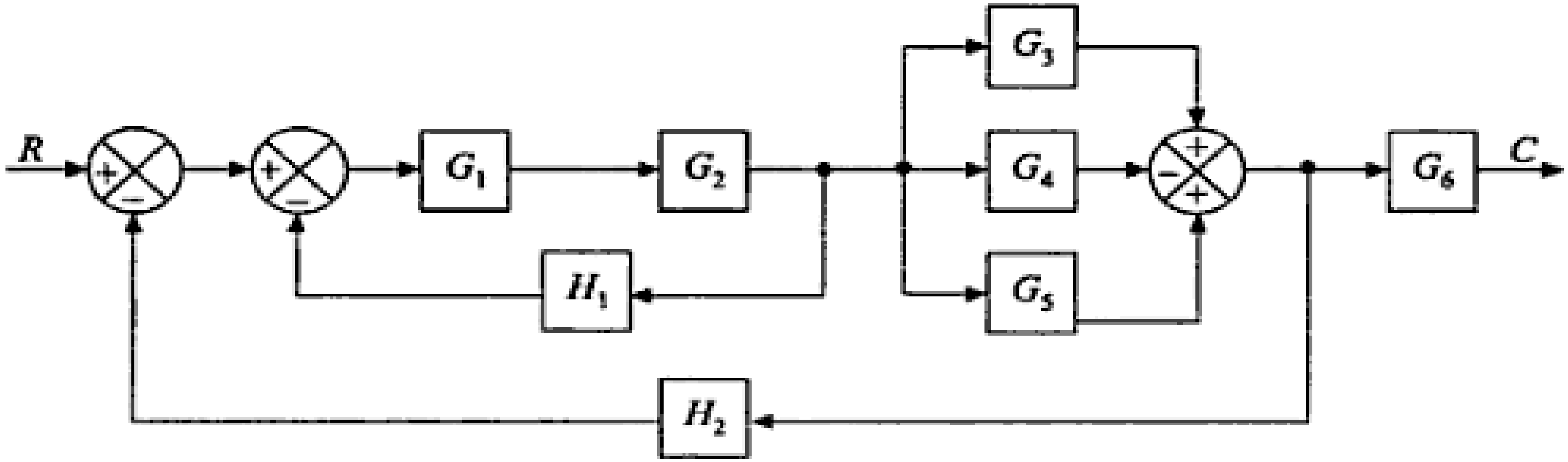




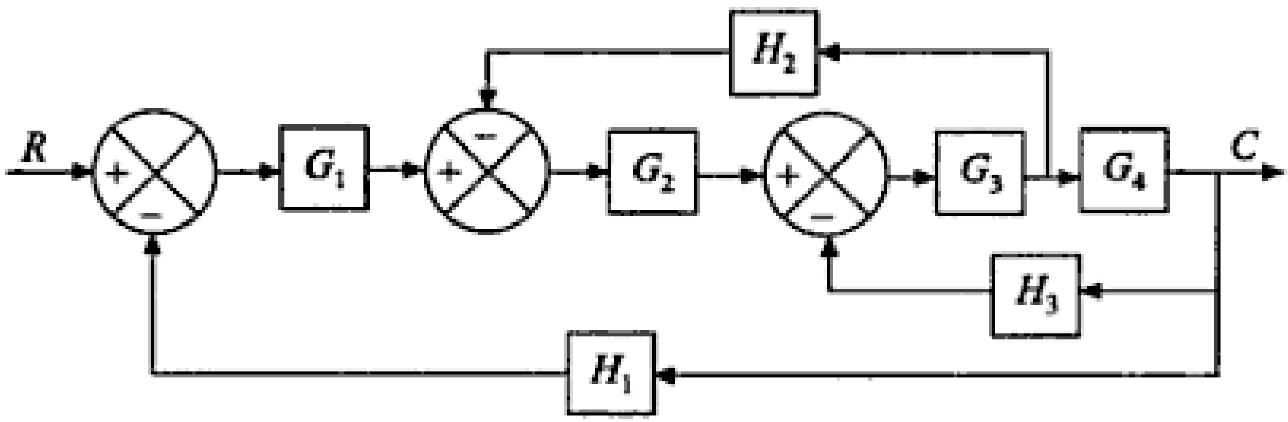
The transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_3}{1 + (G_1G_2 + G_3)H}$$

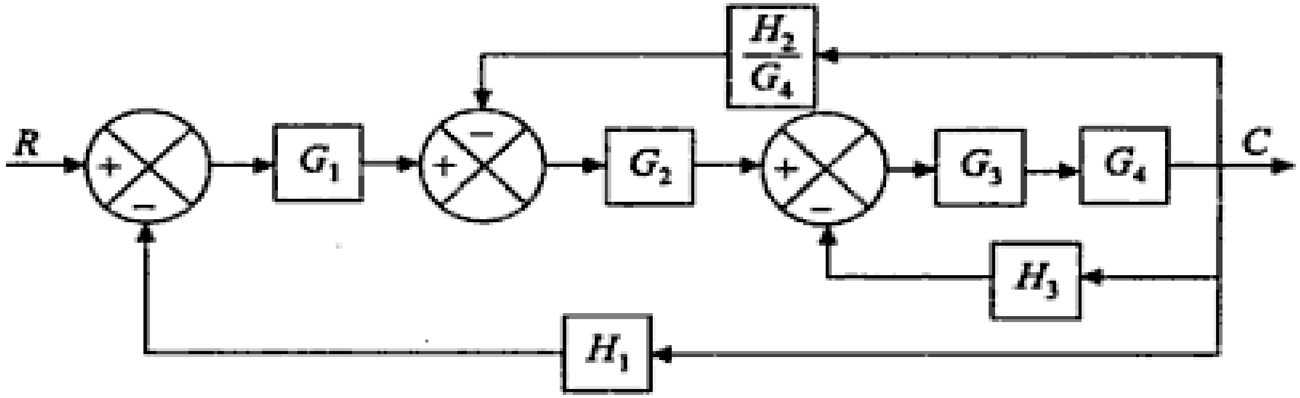
(2)

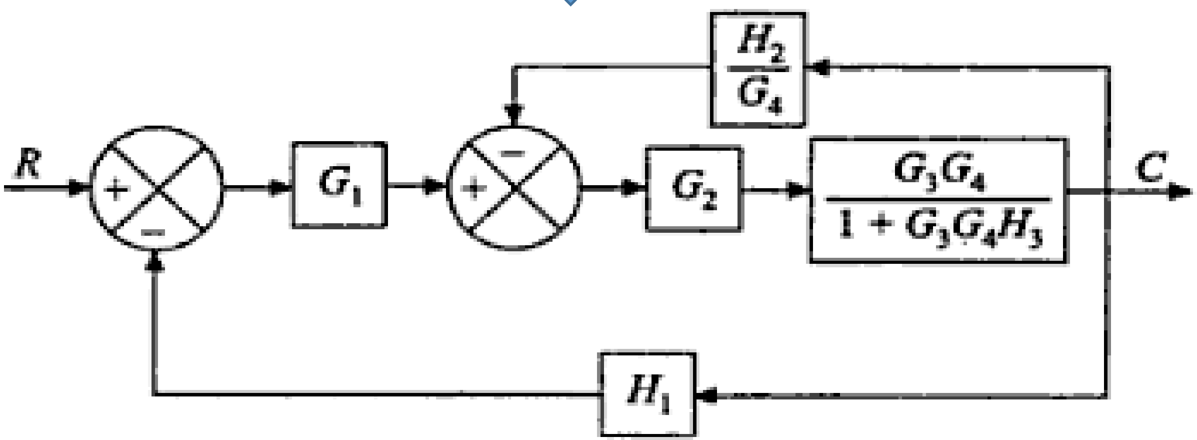


(3)



Solution 3: By inspection, we observe that if the take-off point after G_3 can be moved to that after G_4, the diagram will result in 3 feedback loops. Figure (a) shows how the point is moved to the desired point.





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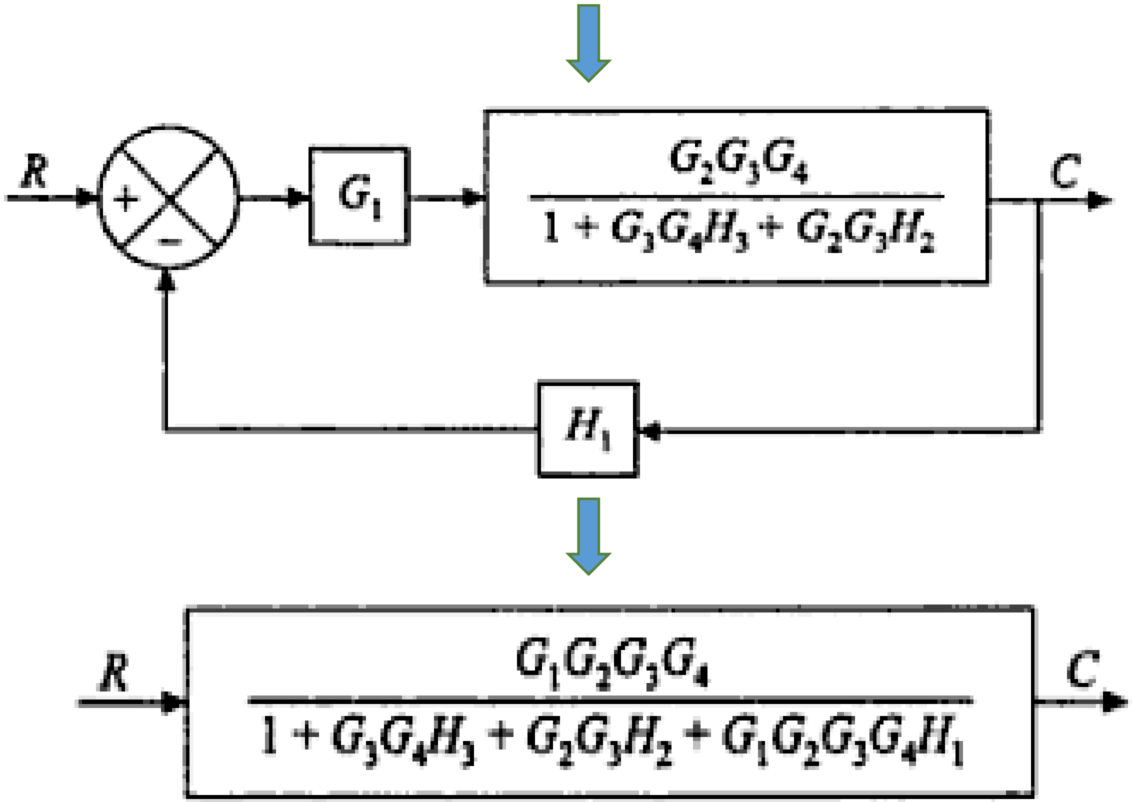
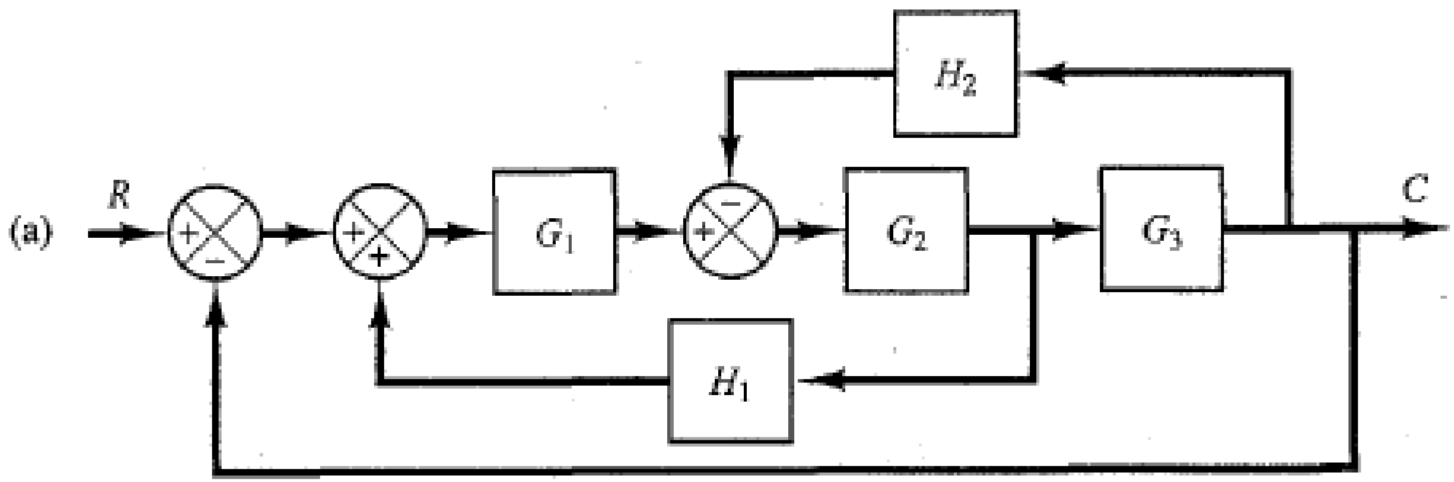


Hence,

$$\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_3G_4H_3+G_2G_3H_2+G_1G_2G_3G_4H_1}$$

(4) Consider the system shown in Figure (a). Simplify the diagram.

- By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1, we obtain Figure (b). Eliminating the positive feedback loop.



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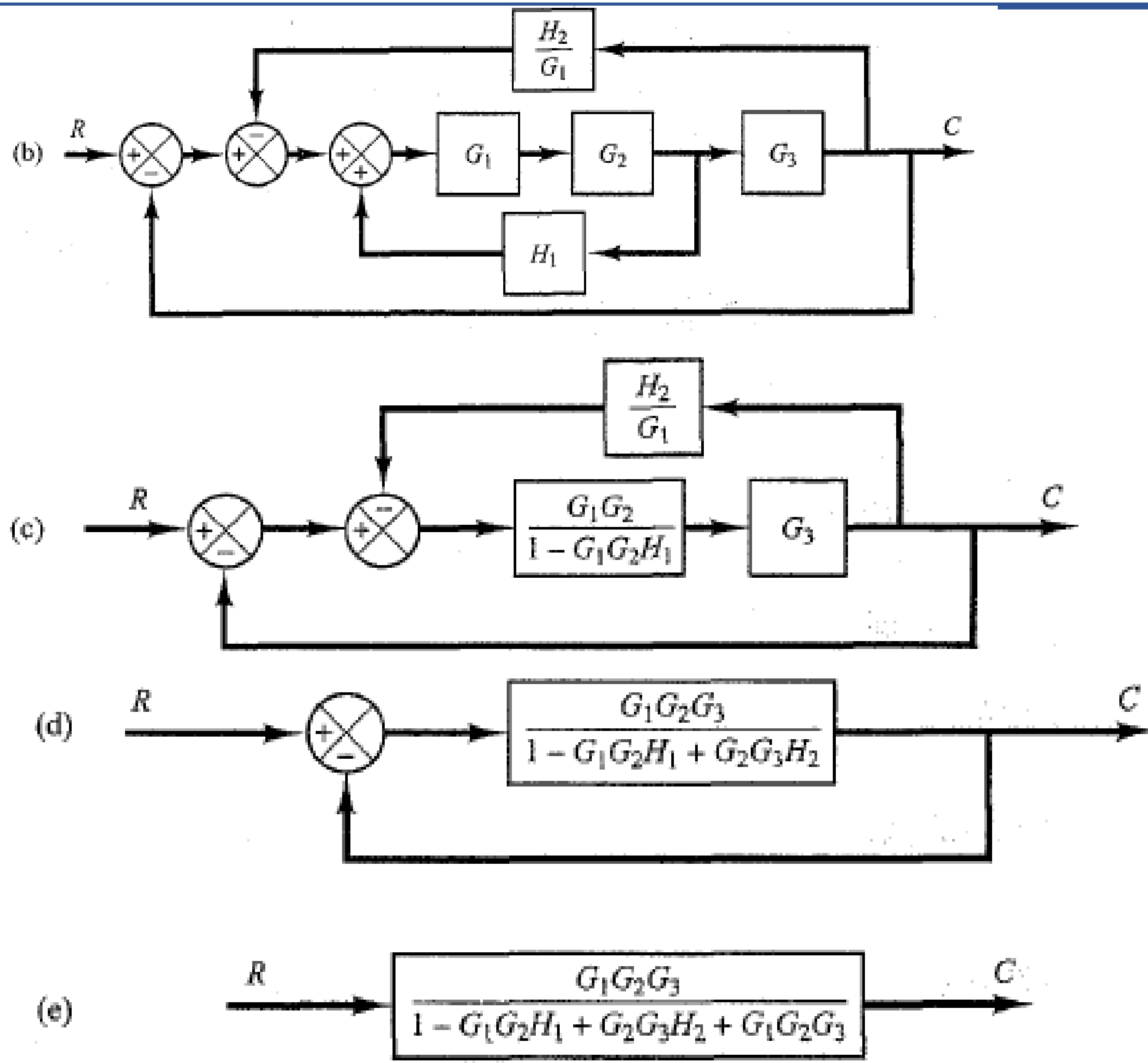

(b)
R
C
G_1
G_2
G_3
H_1
$\frac{H_2}{G_1}$
(c)
R
C
$\frac{G_1G_2}{1 - G_1G_2H_1}$
G_3
$\frac{H_2}{G_1}$
(d)
R
C
$\frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2}$
(e)
R
C
$\frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$

Mathematical Modelling of Physical Systems

Definition:

- A physical system is a collection of physical objects connected in some designed pattern to serve some prescribed objective.

- Based on types of motion mechanical systems are classified into two types:

  1. Translational mechanical system

  2. Rotational mechanical system

- The motion of the body/object during *translational motion is along a straight line or curved path*; whereas during rotational motion, *the motion of an object is about its own axis.*

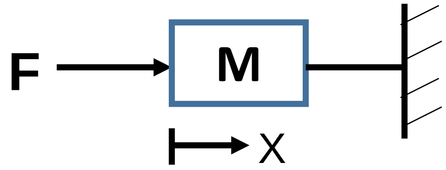
1. Translational mechanical systems

- There are three basic elements involves in the analysis of translational motion. These are:

(i) Mass (ii) Spring (iii) Damper or dash pot

Cont'd…

Mass:

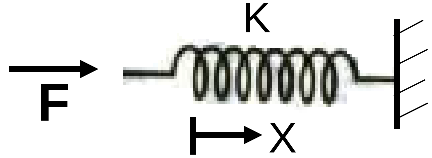


$f_m \propto a$

$$F = f_m = Ma = M\frac{d^2x}{dt^2}$$

Spring: let the linear spring constant for the spring be K. in this case the spring is subjected to force and it undergoes elastic deformation. The relation is $f_k \propto$ displacement.

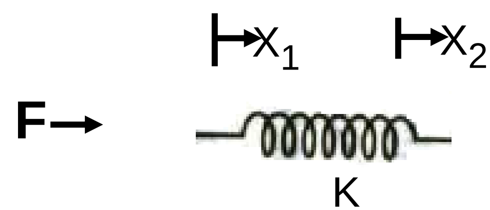
- If one end is fixed to the reference



$F_k \propto x$

$F = F_k = kx$

- If both ends are free



$F = F_k = k(x_1 - x_2)$

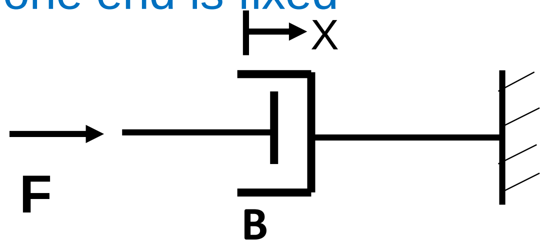
Cont'd…

Damper: Motion is opposed by friction. Types of frictional forces are:

(**i) Coulomb frictional force:** sliding friction b/n dry surface

(ii) Viscous friction force (F_b) : friction b/n moving surface by a viscous fluid or friction b/n solid body and a fluid medium.

- If one end is fixed



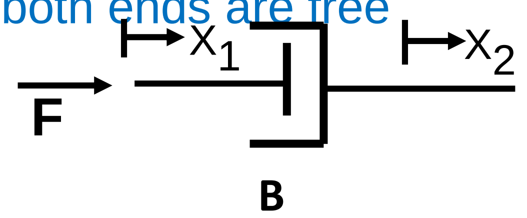
$F_b \propto V$ *(Velocity)*

$F_b = BV$

$$F_b = F = B\frac{dx}{dt}$$

Where: B is viscous friction constant

- If both ends are free



$$F_b \propto B\frac{d(x_1 - x_2)}{dt}$$

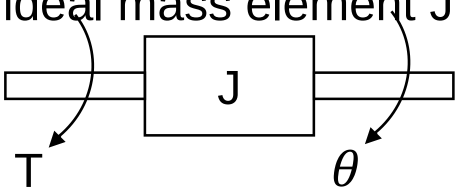
$$F = F_b = B\frac{d}{dt}(x_1 - x_2)$$

2. Rotational Mechanical Systems

- Considering torque (T) and angular displacement (θ) here for object rotating about its own axis.

- The three elements of rotational motion are:
  - Moment Inertia of mass (J)
  - Damper (B)
  - Torsional Spring (K)

Mass: For any ideal mass element J

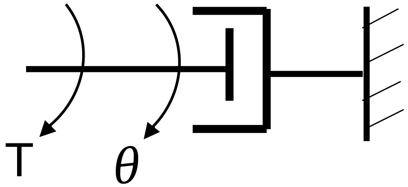


$$T_J \propto \frac{d^2\theta}{dt^2}$$

$$T = T_J = J\,\frac{d^2\theta}{dt^2}$$

Cont'd…

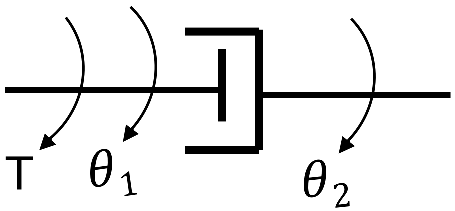
Damper: Opposing torque (T_b) due to friction of dash pot or damper is:



- If one end is fixed.

$$T_b \propto \frac{d\theta}{dt} \quad \Rightarrow \quad T = T_b = B\,\frac{d\theta}{dt}$$

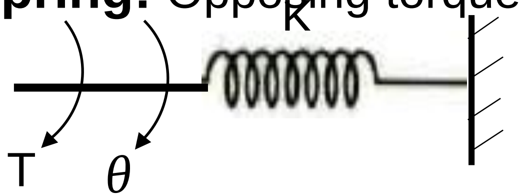
- If both ends are free.



$$T_b = B\,\frac{d\theta}{dt} \quad \Rightarrow \quad T = T_b = B\,\frac{d}{dt}(\theta_1 - \theta_2)$$

Since T is at θ_1, therefore, $\theta = \theta_1 - \theta_2$

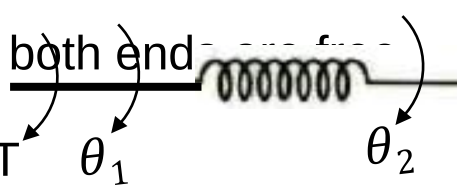
Spring: Opposing torque force due to elasticity of spring K is



$T_K \propto \theta$

Total torque ,$T = T_K = K\theta$

- If both ends are free



$T = T_K = K\,(\theta_1 - \theta_2)$

Modelling of Electrical Systems

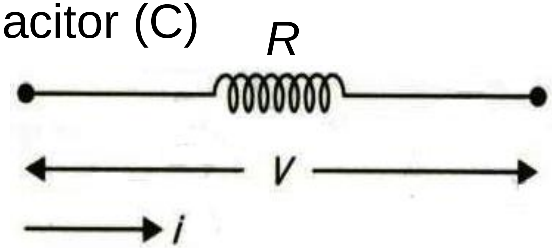
- Basic elements of electrical system are:

  (i) Resistor (R)

  (ii) Inductor (L)

  (iii) Capacitor (C)

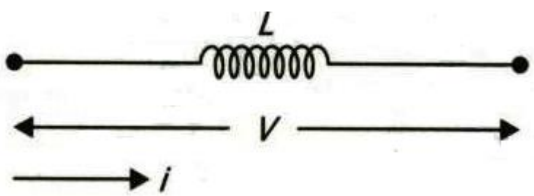
Resistor:



$$V = R\frac{dQ}{dt} = iR$$

Where *V* is voltage, Q is charge and ***i*** is current

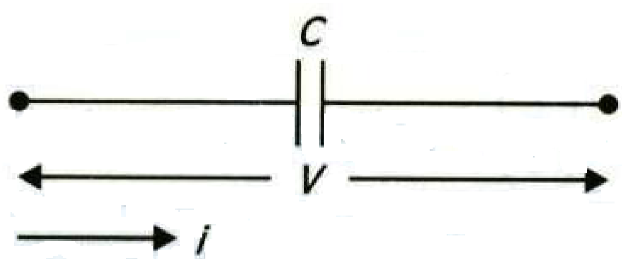
Inductor:



$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_0^t V\,dt$$

Capacitor:



$$V = \frac{Q}{C} \Rightarrow v = \frac{1}{C}\int_0^t i\,dt$$

$$i = C\frac{dv}{dt}$$

Analogous Systems

- Two different physical systems having same mathematical model
- In analogous system a non – electrical systems is expressed in terms of its electrical counter part.
- There are two methods to get analogous systems:

(i) Force - voltage analogy (ii) Force - current analogy

1) Translational mechanical system to Electrical System

(i) Force – Voltage analogy

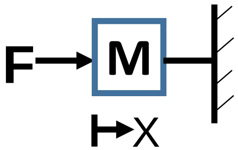
Mechanical system	Electrical system
Input: Force	Input: voltage source
Output: Velocity	Output: current through element

Force – Voltage analogy Cont'd…

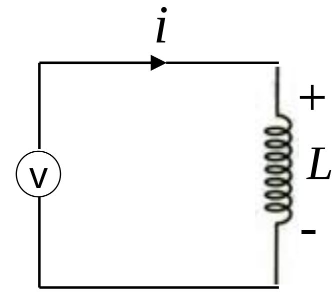
Mechanical system

Electrical system

1)

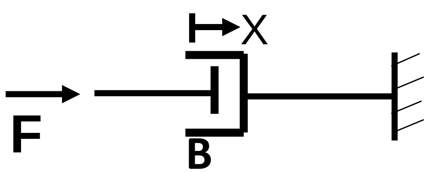


$$F = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

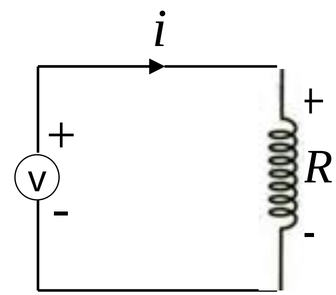


$$v = L\frac{di}{dt}$$

2)

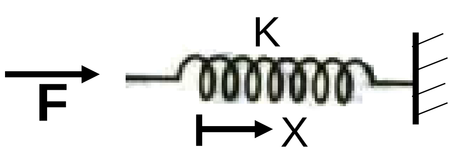


$$\mathrm{F} = \mathrm{B}\frac{dx}{dt} = BV$$

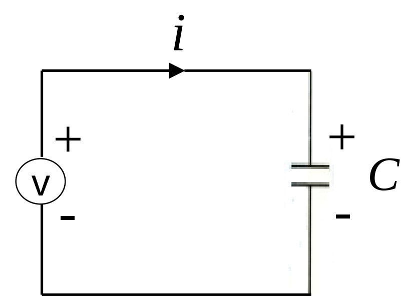


$$v = iR$$

3)



$$\mathrm{F} = \mathrm{kx} = \mathrm{k}\int v\,dt$$



$$v = \frac{1}{C}\int v\,dt$$

Cont'd…

- **Note:**

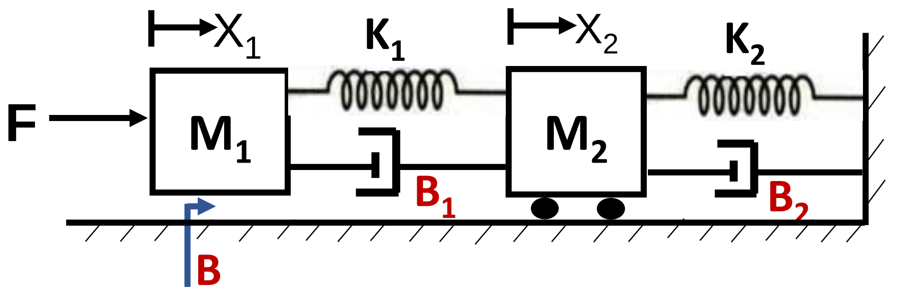
  i. In mechanical system the elements having same velocity are said to be in series, similarly in electrical system the elements in series will have same current

  ii. Each node (mass) in mechanical system corresponds to a closed loop in electrical system.

  iii. Number of meshes in electrical system is equal to number masses in mechanical system

  iv. The element connected between two masses in mechanical system is represented as a common element between two meshes in electrical analogous system.

Example: Element between two meshes in electrical analogous system



Mechanical system

Input: Force

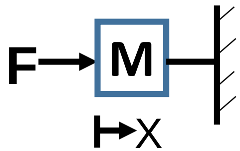
Output: Velocity

Electrical system

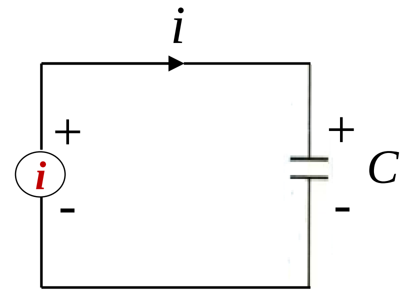
Input: Current source

Output: Voltage across element

1)

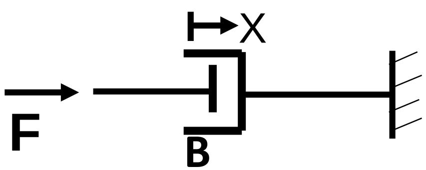


$$F = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

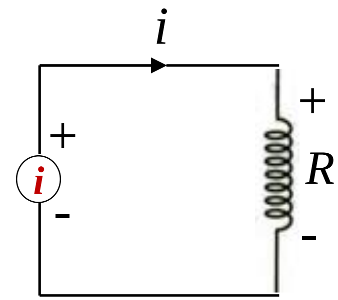


$$\boldsymbol{i} = C\frac{dV}{dt}$$

2)

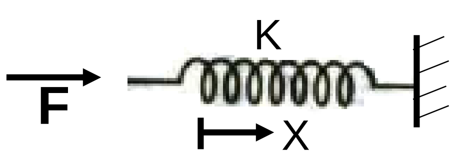


$$\mathrm{F} = \mathrm{B}\frac{dx}{dt} = BV$$

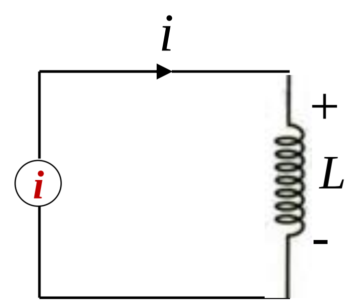


$$\boldsymbol{i} = \frac{V}{R}$$

3)



$$\mathrm{F} = \mathrm{kx} = \mathrm{k}\int vdt$$



$$i = \frac{1}{L}\int Vdt$$

Note:

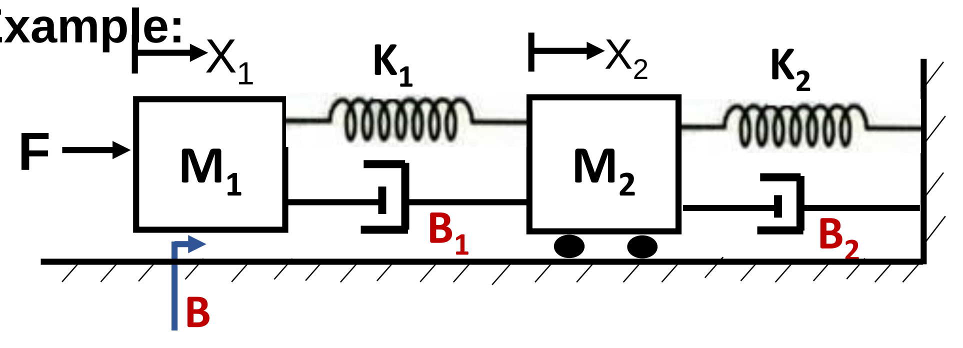
i) In mechanical system the elements in parallel will have same forces similarly, in electrical system parallel elements will have same voltage.

ii) Each mode (mass) in mechanical system corresponds to a node in electrical system.

iii) Number of nodes in electrical system is equal to number of **Mass** in mechanical system.

iv) The element connected b/n two mass in mechanical system is represented as common element b/n nodes in electrical system.

Example:



2) Rotational Mechanical system to Electrical System

(i)Torque – Voltage analogy

Mechanical rotational system

Input: Torque

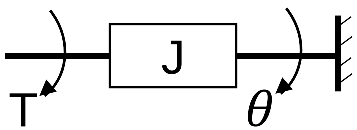
Output: angular velocity

Electrical system

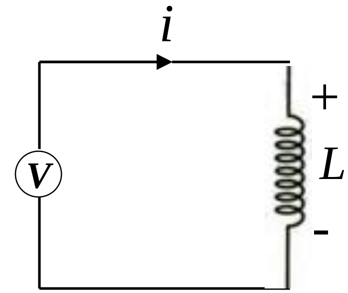
Input: voltage source

Output: current through element

1)

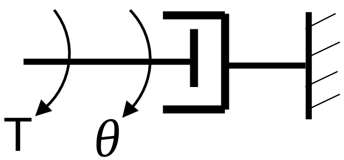


$$T = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$

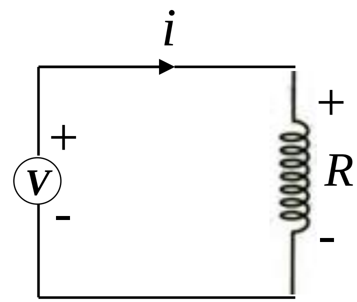


$$V = L\frac{di}{dt}$$

2)

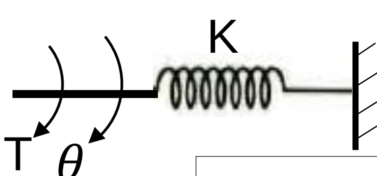


$$T = B\frac{d\theta}{dt} = B\omega$$

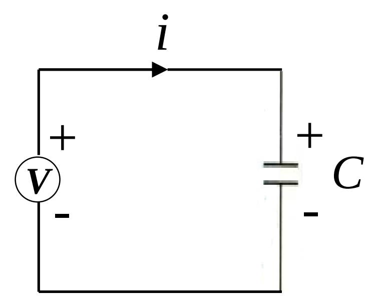


$$V = iR$$

3)



$$T = K\theta = K\int \omega dt$$



$$V = \frac{1}{C}\int idt$$

2) Rotational Mechanical system to Electrical System

(ii)Torque – Current analogy

<u>Mechanical rotational system</u>

Input: Torque

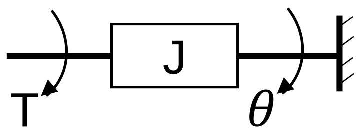
Output: Angular velocity

<u>Electrical system</u>

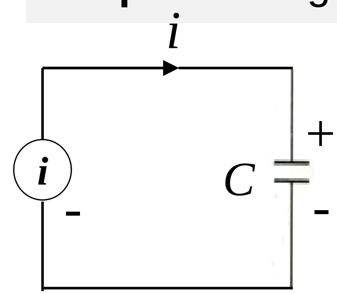
Input: Current source

Output: Voltage across the element

1)

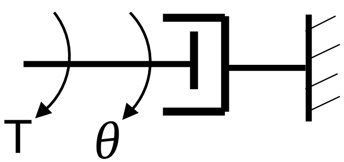


$$T = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$

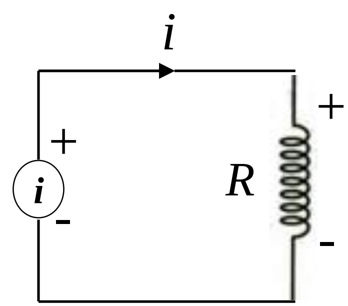


$$i = C\frac{dV}{dt}$$

2)

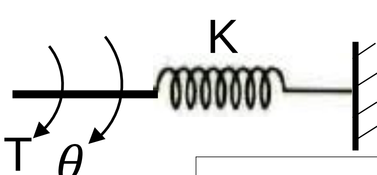


$$T = B\frac{d\theta}{dt} = B\omega$$

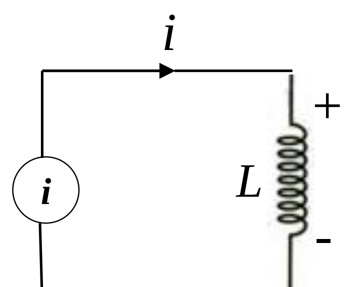


$$i = \frac{V}{R}$$

3)



$$T = K\theta = K\int \omega dt$$



$$i = \frac{1}{L}\int V dt$$

Summary: Counterparts of Translational and Rotational Motion

Force-voltage Analogy

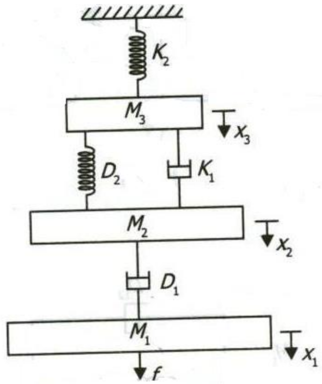
Translational	Electrical	Rotational
Force (f)	Voltage (V)	Torque (T)
Mass (M)	Inductance (L)	Inertia (J)
Damper (D)	Resistance (R)	Damper (D)
Spring (K)	Elastance $\left(\frac{1}{C}\right)$	Spring (K)
Displacement (x)	Charge (q)	Displacement (θ)
Velocity (u)	Current (i)	Velocity (ω)

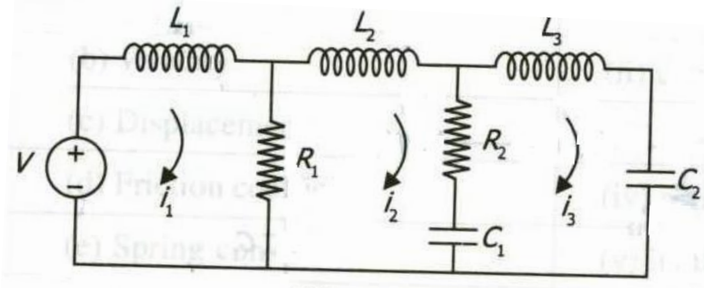
Force-current Analogy

Translational	Electrical	Rotational
Force (f)	Current (i)	Torque (T)
Mass (M)	Capacitance (C)	Inertia (J)
Spring (K)	Reciprocal of Inductance $\left(\frac{1}{L}\right)$	Spring (K)
Damper (D)	Conductance $\left(\frac{1}{R}\right)$	Damper (B)
Displacement (x)	Flux Linkage (ψ)	Displacement (θ)
Velocity $\left(u = \frac{dx}{dt}\right)$	Voltage (V) $= \frac{d\psi}{dt}$	Velocity $\left(\omega = \frac{d\theta}{dt}\right)$

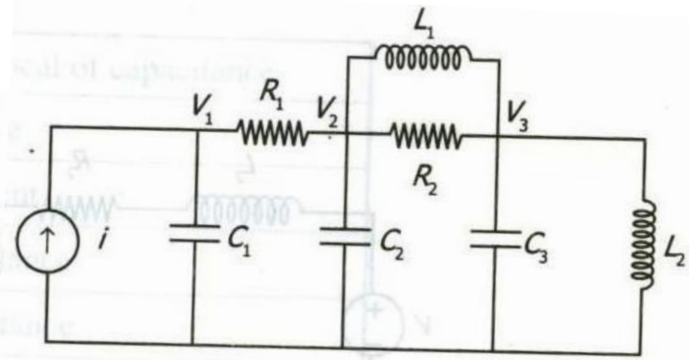
Examples: 1. Obtain the analogous electrical network of Fig below.

Using (a) *f-v* and (b) *f - I* analogy



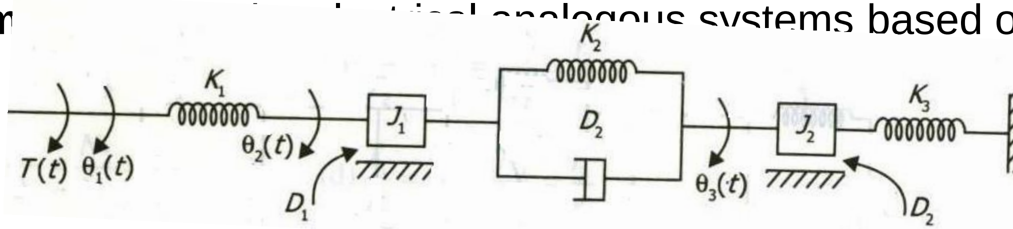


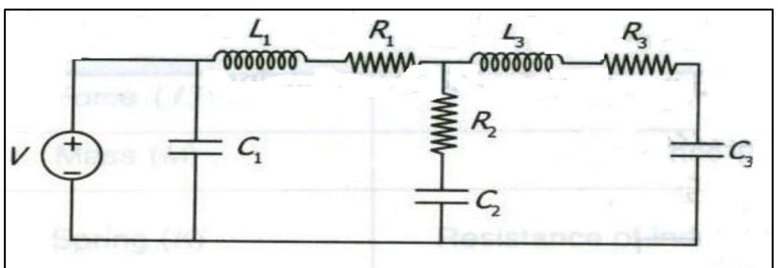
f-V analogy



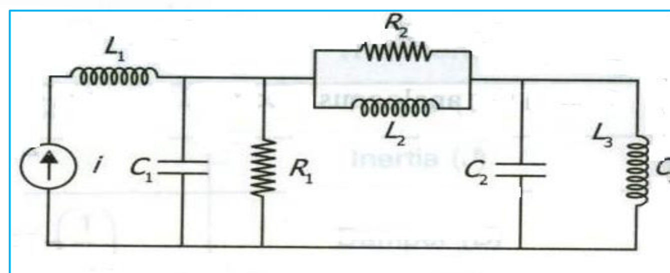
f-i analogy

Exam… analogous systems based on (a) *T-v* and (b) *T- i* analogy





T− V analogy

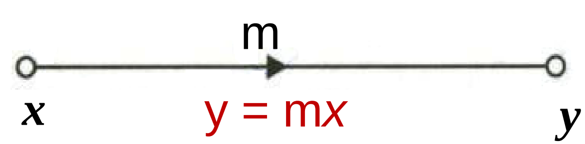


T− i analogy

Reading Assignment

1. Thermal system modeling
2. Hydraulics system modeling

- For complicated systems, BD reduction process will be tedious and time consuming .
- In such cases SFG is an alternative method and is developed by S.J. Mason.
- Signal Flow Graph (SFG) - is graphical representation of a control system.
- In ***which nodes represent the system variables*** and directed ***branches b/n the nodes represent the functional r/ship b/n the variables***.
- Consider the basic SFG shown in fig below*. It consists of two nodes x and y connected by a single branch directed from x node to y node.* The node x and y represent two variable and the directed branch b/n them represents the following r/ship.



> It is important to note that the above SFG does not imply, x = y/m. Further it is to be noted that if the branch is directed from the node y towards x in the fig, it does not imply y = - mx, but it means x = my. Usually the - ve sign is associated with the functional relationships of m.

Definitions of Basic Terms for SFG

- The element of a signal flow graph are given in table below and they are pictorial represented in Fig. 3.26 as well.

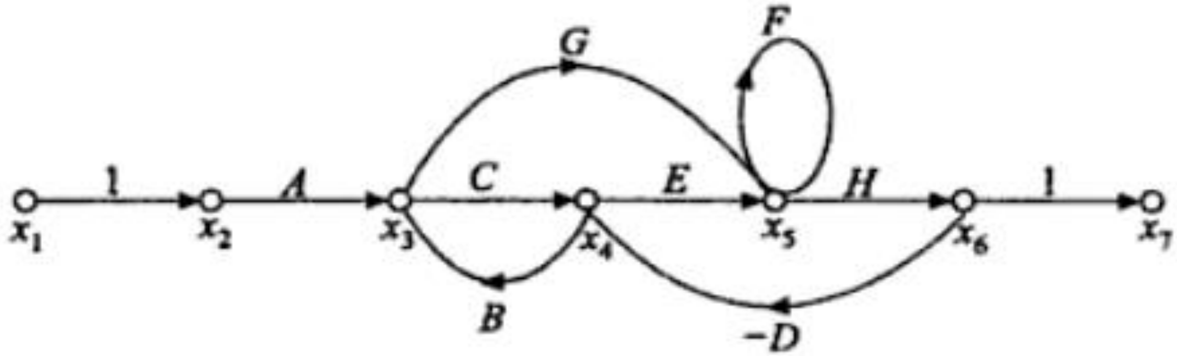


Figure 3.26 A signal flow graph showing different elements.

Table 3 Signal flow graph elements

Element	*What It Is*
Node	A node represents a dependent or independent variable. $x_1, x_2, ..., x_7$ in Figure 3.26 are nodes. Nodes can be divided into the following three categories: 1. Source (or Input) node—node having only outgoing branches (x_1). 2. Sink (or Output) node—node having only incoming branches (x_7). 3. Chain node—node having both outgoing and incoming branches (x_2, x_3, x_4, x_5, x_6).

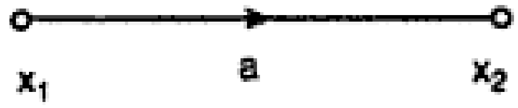
Cont'd…

Branch	Lines joining nodes are branches. An arrow on the branch indicates the direction of flow of the signal, and the number written on it indicates the *gain* of the branch. In Figure 3.26, $x_3 = Ax_2$, where A is the gain of the branch.
Forward path	All paths leading to output and originating at the input are forward paths. If we denote forward paths in Figure 3.26 by p_1 and p_2 they are: $p_1 : x_1x_2x_3x_4x_5x_6x_7$ $\quad p_2 : x_1x_2x_3x_5x_6x_7$

Loop or feedback path	A path originating from and terminating at the same node, without crossing any other node twice, is a loop. There are four loops in Figure 3.26, namely $L_1 : x_3x_4x_3$ $\quad L_2 : x_4x_5x_6x_4$ $\quad L_3 : x_5x_5$ $\quad L_4 : x_3x_5x_6x_4x_3$. Of these, L_3 is a self-loop.

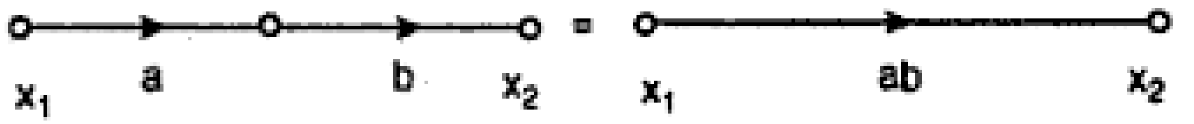
Forward path gain	The product of all gains associated with a forward path is the forward path gain. Gains for two paths are: $p_1 : ACEH$ $\quad p_2 : AGH$
Loop gain	The product of gains associated with a loop is the loop gain. Gains for four loops are: $L_1 : BC$ $\quad L_2 : -EHD$ $\quad L_3 : F$ $\quad L_4 : -GHDB$
Non-touching loops	Loops having no common nodes are non-touching loops. L_1 and L_3 is a pair of non-touching loops.

Rule 1. (i)

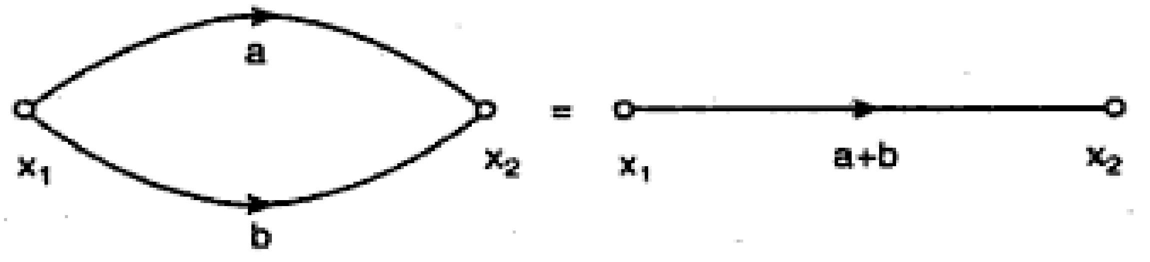


$x_2 = a\,x_1$

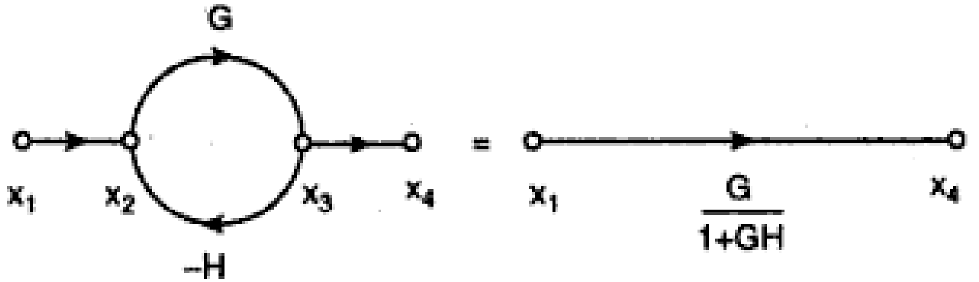
Rule 2.



Rule 3.



Rule 4.



Mason's Gain Formula

- The transfer function of a system *(overall system gain)* can be determined from signal flow graph by using Mason's Gain Formula
- Mason's gain formula is given by:

$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} p_i \Delta_i}{\Delta}$$

where N = Total number of forward paths

p_i = Gain of the ith forward path

Δ = Determinant of the graph

= 1 − (Σ all individual loop gains)
+ (Σ gain products of all possible combinations of *two* non-touching loops)
− (Σ gain products of all possible combinations of *three* non-touching loops)
+ ...

Δ_i = Path-factor for the ith path

= 1 − (part of Δ that is non-touching to the path p_i)

= 1 – Loop gains which are non touching i^{th} forward path

Example. Find transfer function of fig 3.26

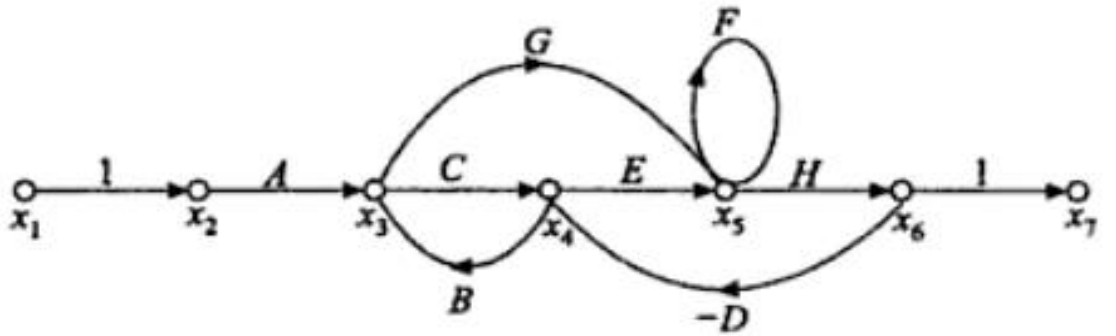


Figure 3.26 A signal flow graph showing different elements.

We can find out the transfer function of signal flow graph of Figure 3.26 from the following data (see Table 3.3 for clarification):

$$N = 2$$
$$p_1 = ACEH$$
$$p_2 = AGH$$
$$\Delta = 1 - (BC - EHD + F - GHDB) + BCF$$
$$\Delta_1 = 1 \text{ (since all loops are touching } p_1)$$
$$\Delta_2 = 1 \text{ (since all loops are touching } p_2)$$

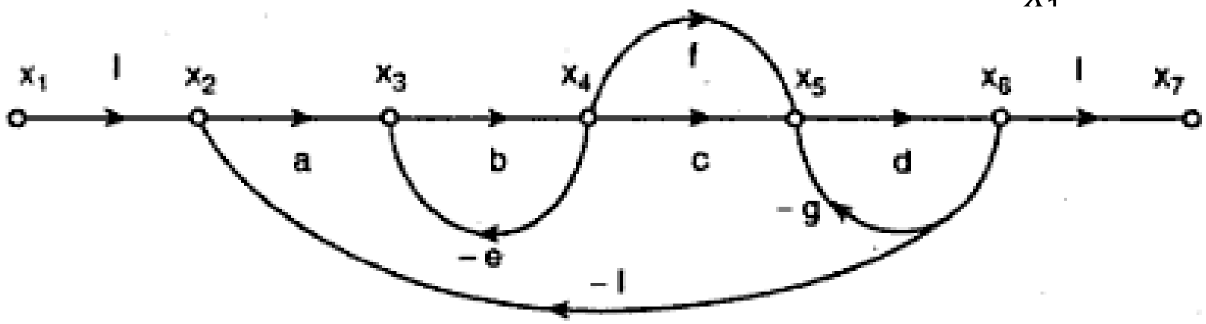
$$\text{Transfer function} = \frac{p_1\Delta_1 + p_2\Delta_2}{\Delta}$$

$$= \frac{ACEH + AGH}{1 - BC + EHD - F + GHDB + BCF}$$

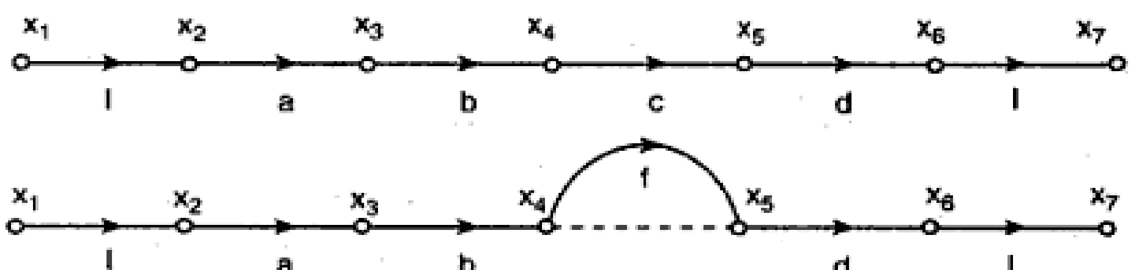
$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} p_i\Delta_i}{\Delta}$$

Cont'd…

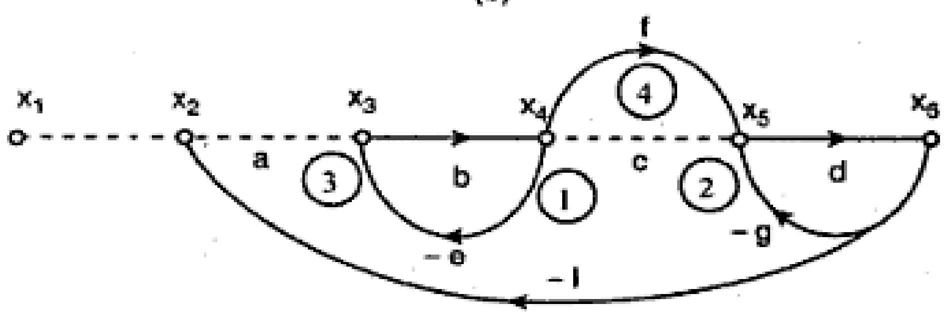
Example 2: For the signal flow graph shown in Fig (a) Find $\frac{X_7}{X_1}$



Solution



(b)



(c)

1. There are two forward paths as shown in Fig. 4.26(b)
   The first forward path gain $T_1 = abcd$
   The second forward path gain $T_2 = abfd$
2. There are four individual feedback loops and they are shown in Fig. 4.26(c). The individual loop gains are,

$$L_1 = -be$$
$$L_2 = -dg$$
$$L_3 = -abcd$$
$$L_4 = -abfd$$

3. There are two non-touching loops L_1 and L_2. Hence the gain product L_1 and L_2 is $bedg$ (the only combination, the other loops being in touch with each other),

$$L_1 L_2 = bedg$$

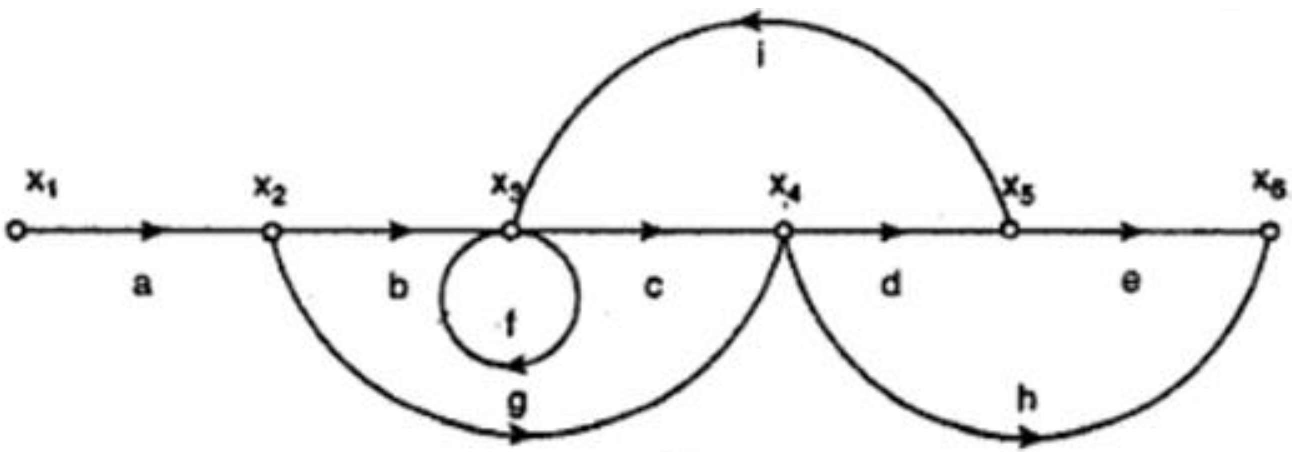
Hence,
$$\Delta = 1 - \sum_n P_{n1} + \sum_n P_{n2} = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_2$$
$$\Delta = 1 + be + dg + abcd + abfd + bedg$$

4. All the feedback loops are in touch with the forward path in both the cases of T_1 and T_2. Hence when their contacts with these forward paths are removed, the loops are broken. $\Delta_1 = \Delta_2 = 1$.

$$A = \frac{x_7}{x_1} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{(abcd + abfd)}{(1 + be + dg + abcd + abfd + bedg)}$$

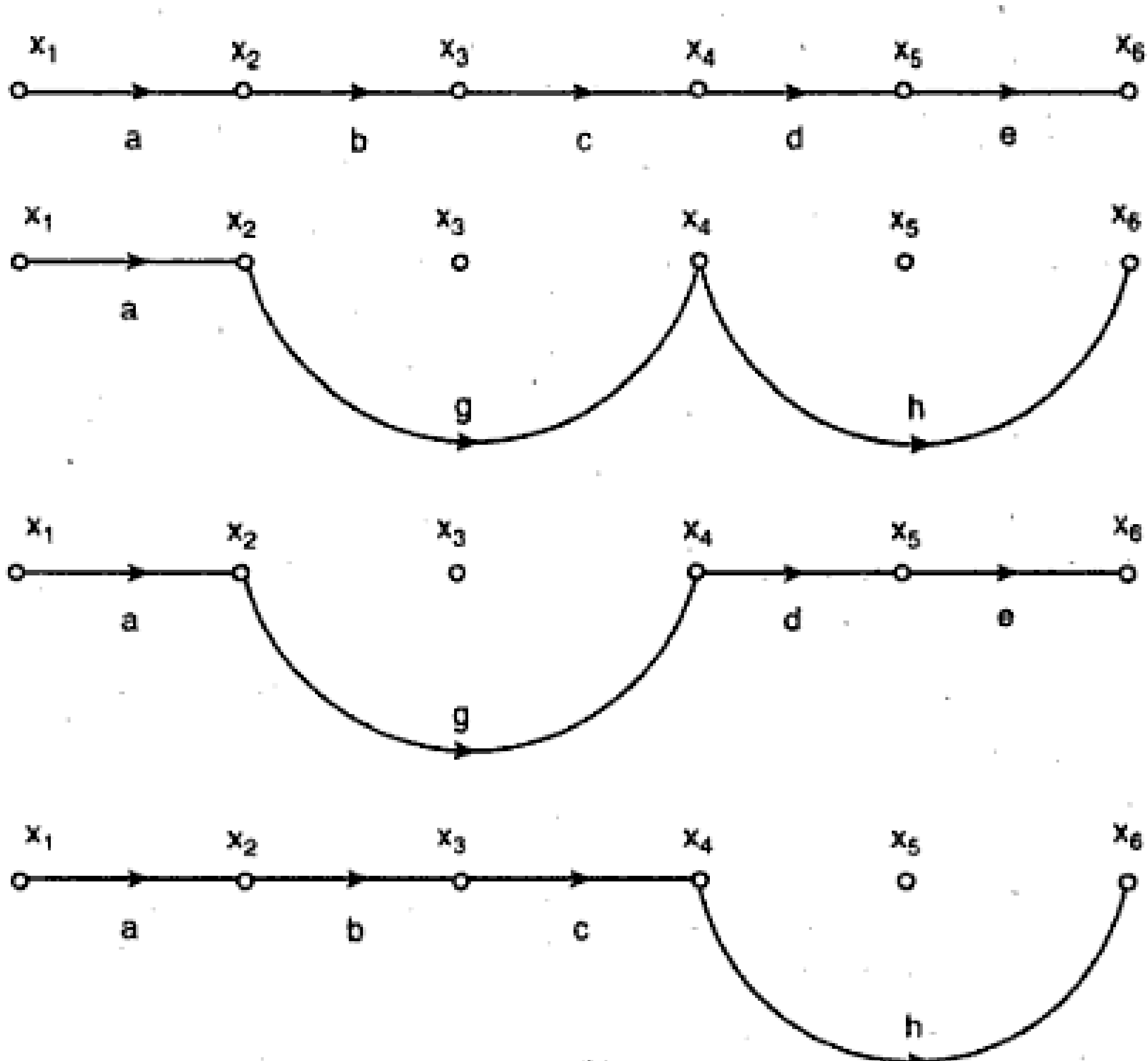
Cont'd…

Example 3: For the signal flow graph shown in Fig (a) Find $\frac{X_6}{X_1}$



(a)

Solution



(b)

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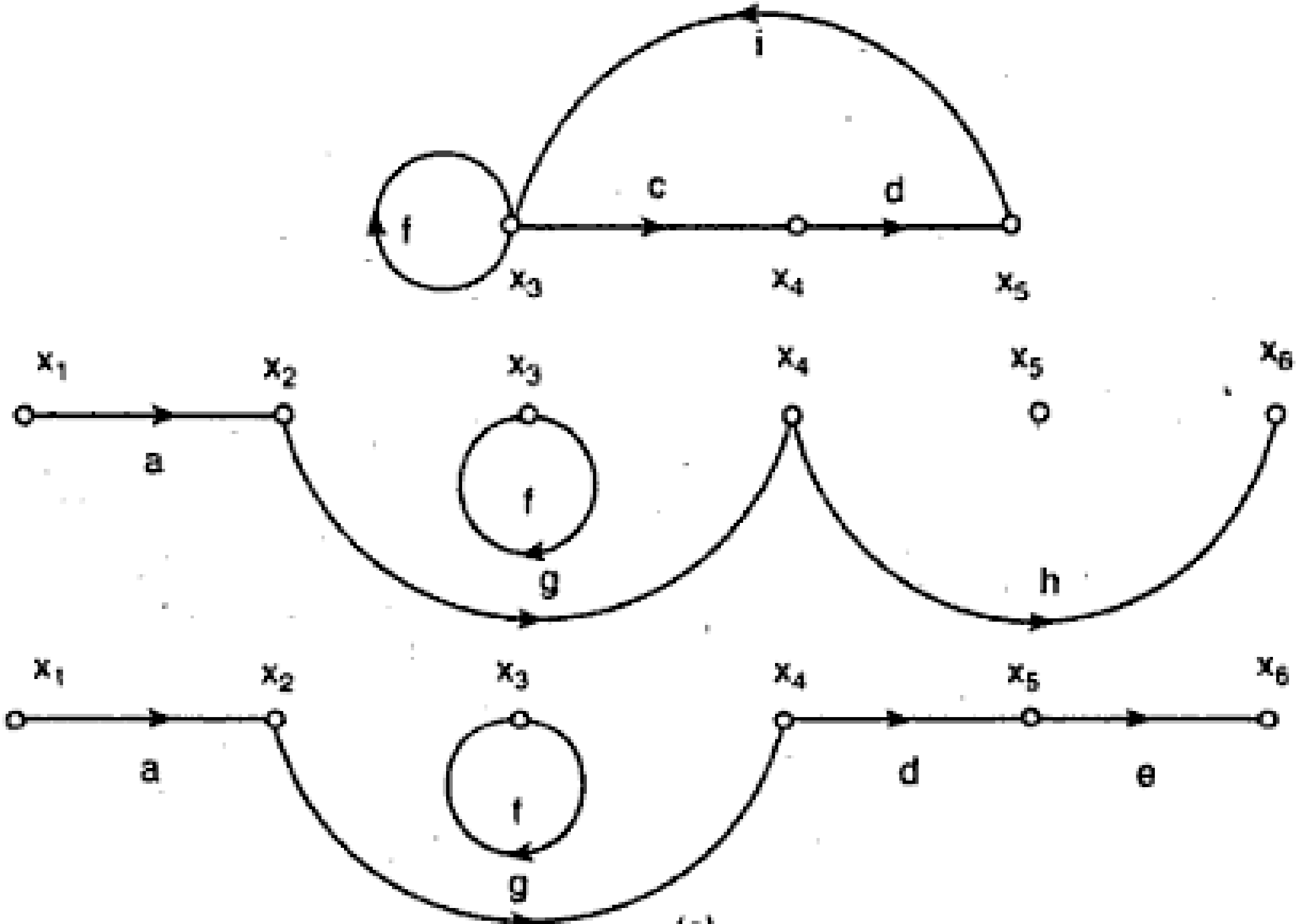


(c)

1. There are four forward paths and they are shown in Fig. 4.25(b). The forward path gains are

$$T_1 = abcde$$
$$T_2 = agh$$
$$T_3 = agde$$
$$T_4 = abch$$

2. There are two individual feedback loops as shown in Fig. 4.25(c). The gains of individual loops are

$$L_1 = f$$
$$L_2 = cdi$$

3. There is no two and three non-touching loops. Hence $\sum_n P_{n2}, \sum_n P_{n3}$ etc. are zero. Hence,

$$\Delta = 1 - \sum_n P_{n1} = 1 - (L_1 + L_2) = 1 - (f + cdi)$$

4. For the forward path gain T_1 there is no non-touching loop with the forward path. The two feedback loops are touching the forward path. Hence $\Delta_1 = 1$. For the forward path with T_2 gain one feedback loop is not touching the forward path as shown in Fig. 4.25(c). Hence Δ_2 for the above loop is found with the usual procedure. Gain product of the loop is f. Other things are zero. Hence $\Delta_2 = 1 - \text{sum of individual loop gain} = (1-f)$. For the forward path with gain T_3 there is one non-touching loop with the forward path as shown in Fig. 4.25(c). This is similar to Δ_2. Hence $\Delta_3 = (1-f)$. For the forward path T_4, both the feedback loops are touching the forward path. Hence, if the connections of these loops with the forward path are removed, both loops are broken. Hence $\Delta_4 = 1$.

The total gain,

$$A = \frac{x_6}{x_1}$$

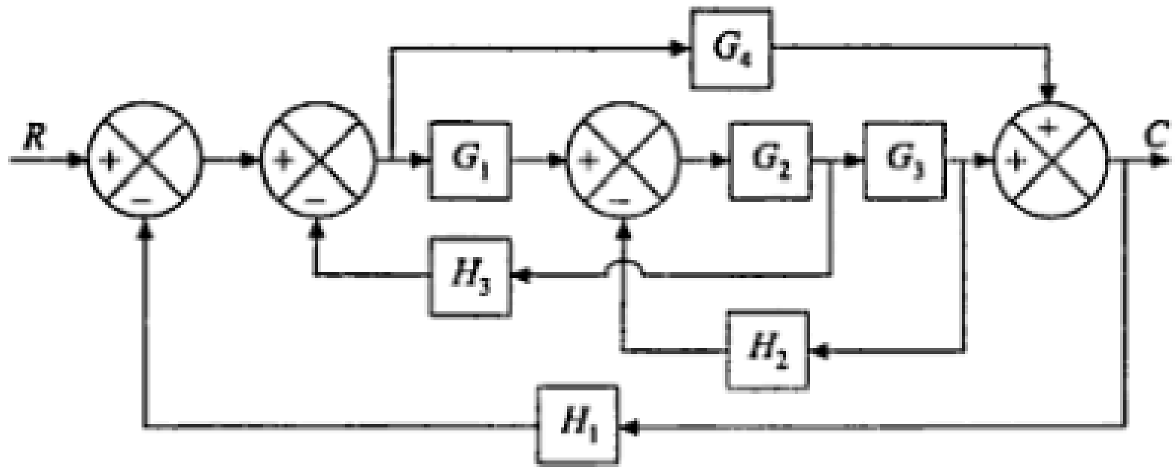
$$= \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

$$\boxed{\frac{x_6}{x_1} = \frac{abcde + agh(1-f) + agde(1-f) + abch}{1-(f+cdi)}}$$

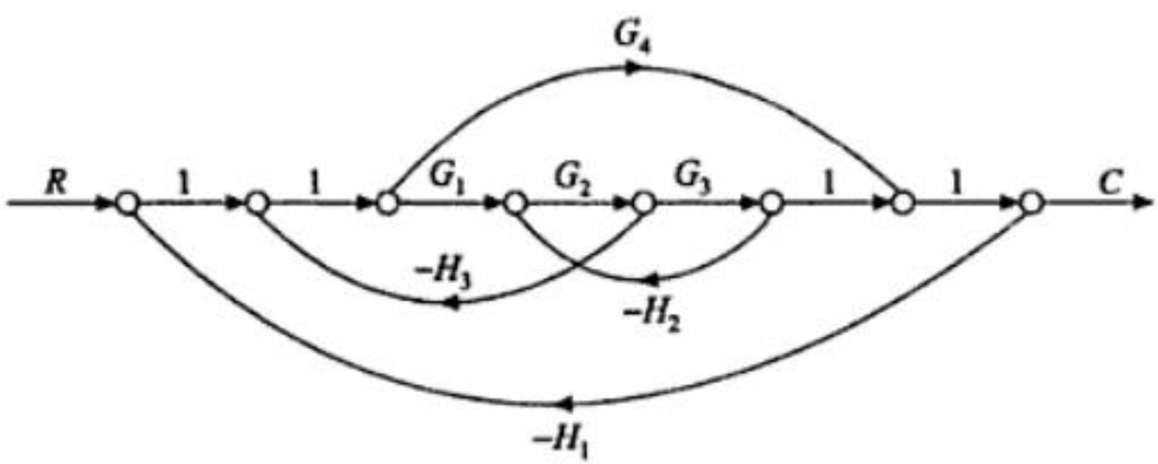
Procedure for converting block diagram to SFG

1. Assume nodes at the input, output, at every summing point, at every branch point and in between cascaded blocks
2. Draw the node separately as small circles and number the circles in the order 1,2,3…
3. From the block diagram find the gain b/n each node in the main forward path and connect all the corresponding circles by straight lines and mark the gain on that nodes.
4. Draw the field forward paths b/n various nodes and mark the gain of field forward paths along with sign.
5. Draw the field back path b/n the various nodes and mark the gain of feed back paths along with sign.

Examples: Find the overall transfer function using Mason's gain formula of the system whose block diagram is shown as follow:



Solution: The signal flow graph of the block diagram is



Here, forward paths and gains are

$$p_1 = G_1G_2G_3$$
$$p_2 = G_4$$

Loops and gains are

$$L_1 = -G_1G_2H_3$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3H_1$$
$$L_4 = -G_4H_1$$

The non-touching loop-pair and gain are $L_2L_4 = G_2G_3G_4H_1H_2$

All loops and pairs touch p_1 but L_2 does not touch p_2. Therefore,

$$\Delta_1 = 1$$
$$\Delta_2 = 1 + G_2G_3H_2$$
$$\Delta = 1 + G_1G_2H_3 + G_2G_3H_2 + G_1G_2G_3H_1 + G_4H_1 + G_2G_3G_4H_1H_2$$

Transfer function $= \dfrac{C(s)}{R(s)}$

$$= \frac{G_1G_2G_3 + G_2G_3G_4H_2 + G_4}{1 + G_1G_2H_3 + G_2G_3H_2 + G_1G_2G_3H_1 + G_4H_1 + G_2G_3G_4H_1H_2}$$

End of Chapter

Thank you!